Mathematical Equations

Equations in Unit 1 — Forces and Motion: Dynamics		
Equation	**Variable**	**Name**
$\vec{F} = m\vec{a}$	$\vec{F}$ = net force m = mass $\vec{a}$ = acceleration	Newton's second law
$\lvert\vec{F}_{\text{f}}\rvert = \mu_{\text{s}}\lvert\vec{F}_{\text{N}}\rvert$ $\lvert\vec{F}_{\text{f}}\rvert = \mu_{\text{k}}\lvert\vec{F}_{\text{N}}\rvert$	$\vec{F}_{\text{f}}$ = force of friction μ_{s} = coefficient of static friction μ_{k} = coefficient of kinetic friction $\vec{F}_{\text{N}}$ = normal force	friction
$\vec{F}_{\text{g}} = m\vec{g}$	$\vec{F}$ = force of gravity at Earth's surface m = mass $\vec{g}$ = acceleration due to gravity (on Earth)	weight
$\Delta d = v\Delta t$ $v_2 = v_1 + a\Delta t$ $\Delta d = v_1\Delta t + \frac{1}{2}a\Delta t^2$ $v_2^2 = v_1^2 + 2a\Delta d$	Δd = displacement v = velocity v_1 = initial velocity v_2 = final velocity a = acceleration Δt = time interval	motion equations (constant acceleration)
$R = \dfrac{v_{\text{i}}^2 \sin 2\theta}{g}$ $H_{\text{max}} = \dfrac{v_{\text{i}} \sin^2 \theta}{2g}$	R = range H_{max} = maximum height v_{i} = initial velocity θ = launch angle g = acceleration due to gravity	projectile range projectile maximum height
$a_{\text{c}} = \dfrac{v^2}{r}$	a_{c} = centripetal acceleration v = velocity r = radius	centripetal acceleration
$F_{\text{c}} = \dfrac{mv^2}{r}$	F_{c} = centripetal force v = velocity r = radius	centripetal force
$\dfrac{T^2}{r^3} = k$ $\dfrac{T_{\text{A}}^2}{r_{\text{A}}^3} = \dfrac{T_{\text{B}}^2}{r_{\text{B}}^3}$	T = period of planet r = average distance from planet to Sun k = constant T_{A} = period of planet A T_{B} = period of planet B r_{A} = average distance of planet A to Sun r_{B} = average distance of planet B to Sun	Kepler's third law
$F_{\text{g}} = G\dfrac{m_1 m_2}{r^2}$	F_{g} = force of gravity between 2 point masses G = universal gravitational constant m_1 = first mass m_2 = second mass r = distance between the centres of the masses	Newton's law of universal gravitation

Equations in Unit 2 — Energy and Momentum		
$\vec{p} = m\vec{v}$	$\vec{p}$ = momentum m = mass $\vec{v}$ = velocity	momentum
$\vec{J} = \vec{F}\Delta t = m\vec{v}_2 - m\vec{v}_1 = \Delta\vec{p}$	$\vec{J}$ = impulse $\vec{F}$ = force Δt = time interval m = mass $\vec{v}_1$ = initial velocity $\vec{v}_2$ = final velocity $\Delta\vec{p}$ = change in momentum	impulse
$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}'_A + m_B\vec{v}'_B$	m_A = mass of object A m_B = mass of object B $\vec{v}_A$ = velocity of object A *before* collision $\vec{v}_B$ = velocity of object B *before* collision $\vec{v}'_A$ = velocity of object A *after* collision $\vec{v}'_B$ = velocity of object B *after* collision	conservation of momentum
$v'_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1$ $v'_2 = \left(\frac{2m_1}{m_1 + m_2}\right)v_1$	v'_1 = velocity of object 1 *after* collision v'_2 = velocity of object 2 *after* collision m_1 = mass of object 1 m_2 = mass of object 2 v_1 = velocity of object 1 *before* collision	perfectly elastic, head-on collision, using a frame of reference such that $v_2 = 0$
$W = F\Delta d \cos\theta$	W = work F = applied force Δd = displacement θ = angle between force and displacement	work
$E_k = \frac{1}{2}mv^2$	E_k = mechanical kinetic energy m = mass v = velocity	mechanical kinetic energy
$E_g = mg\Delta h$	E_g = gravitational potential energy m = mass Δh = change in height	gravitational potential energy
$E_k + E_g + E_e = E'_k + E'_g + E'_e$	E_k = initial kinetic energy E_g = initial gravitational energy E_e = initial elastic energy E'_k = final kinetic energy E'_g = final gravitational energy E'_e = final elastic energy	conservation of mechanical energy
$F_a = kx$	F_a = applied force k = spring constant x = extension or compression of spring	Hooke's law
$E_e = \frac{1}{2}kx^2$	E_e = elastic potential energy k = spring constant x = extension or compression of spring	elastic potential energy
$v = \sqrt{\frac{2GM_p}{r_p}}$	v = escape speed G = universal gravitational constant M_p = mass of planet r_p = radius of planet	escape speed

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$F_c = \frac{mv^2}{r} = \frac{4\pi^2 rm}{T^2}$	F_c = centripetal force m = mass of object v = speed of object r = radius of orbit T = period of orbit	centripetal force
$v = \sqrt{\frac{GM}{r}}$	v = speed of orbiting object G = universal gravitational constant M = mass of planet or star r = radius of orbit	speed of satellite
$a_c = \frac{v^2}{r} = \frac{4\pi^2}{T^2} r$	a_c = centripetal acceleration v = speed of orbiting object r = radius of orbit T = period of orbit	centripetal acceleration
$T = \sqrt{\frac{4\pi^2 r^3}{GM}}$	T = period of orbit r = radius of orbit G = universal gravitational constant M = mass of planet or star	orbital period (squared)
$E_g = -\frac{GMm}{r}$ $E_k = \frac{GMm}{2r}$ $E_{total} = -\frac{GMm}{2r}$ $E_{binding} = \frac{GMm}{r}$	E_g = gravitational potential energy E_k = kinetic energy E_{total} = total orbital energy $E_{binding}$ = binding energy	orbital energies
$F_{thrust} = \left(\frac{m_{gas}}{\Delta t}\right)\Delta v_{gas}$	F_{thrust} = thrust force m_1 = mass of expelled gas Δv_{gas} = speed of expelled gas Δt = time interval	rocket thrust
Equations in Unit 3 — Electric, Gravitational, and Magnetic Fields		
$F_Q = k\frac{q_1 q_2}{r^2}$	F_Q = electrostatic force between charges k = Coulomb's constant q_1 = electric charge on object 1 q_2 = electric charge on object 2 r = distance between object centres	Coulomb's law
$\vec{E} = \frac{\vec{F}_Q}{q}$	$\vec{E}$ = electric field intensity $\vec{F}_Q$ = electric force q = electric charge	electric field intensity
$\vec{g} = \frac{\vec{F}_g}{m}$ $g = \frac{Gm}{r^2}$	$\vec{g}$ = gravitational field intensity $\vec{F}_g$ = force of gravity m = mass G = universal gravitational constant r = distance from centre of object	gravitational field intensity
$\lvert\vec{E}\rvert = k\frac{q}{r^2}$	$\vec{E}$ = electric field intensity k = Coulomb's constant q = source charge r = distance from centre of charge	Coulombic electrostatic field

$V = \frac{E_Q}{q}$ $V = k\frac{q}{r}$	V = electric potential difference E_Q = electric potential energy q = electric charge r = distance k = Coulomb's constant	electric potential electric potential due to a point charge
$\Delta V = \frac{W}{q}$	ΔV = electric potential difference W = work done q = electric charge	electric potential difference
$\lvert\vec{E}\rvert = \frac{\Delta V}{\Delta d}$	$\lvert\vec{E}_Q\rvert$ = electric field intensity ΔV = electric potential difference Δd = component of displacement between points, parallel to field	electric field and potential difference
$F_M = qvB\sin\theta$	F_M = magnitude of the magnetic force on moving charged particle q = electric charge on particle v = speed of particle B = magnetic field intensity θ = angle between velocity vector and magnetic field vector	force on a moving charge in a magnetic field
$F_M = IlB\sin\theta$	F_M = magnitude of the magnetic force on moving charged particle B = magnetic field intensity I = electric current in conductor l = length of conductor in magnetic field θ = angle between conductor and magnetic field	force on a current-carrying conductor in a magnetic field
Equations in Unit 4 — The Wave Nature of Light		
$T = \frac{\Delta t}{N}$ $f = \frac{1}{T}$ $f = \frac{N}{\Delta t}$	T = period f = frequency Δt = time interval N = number of cycles	period and frequency
$PD = (n - \frac{1}{2})\lambda = d\sin\theta$ $n = 1, 2, 3, \ldots$ for dark fringes $PD = n\lambda = d\sin\theta$ $n = 0, 1, 2, 3, \ldots$ for light fringes $\lambda \cong \frac{d}{(n - \frac{1}{2})}\left(\frac{y_n}{x}\right)$ $\lambda \cong \left(\frac{d}{n}\right)\left(\frac{y_n}{x}\right)$	PD = path difference n = nodal line number λ = wavelength d = slit separation θ = angle between central bisector and line formed from slit to nodal point y_n = distance from central maximum fringe to fringe n x = distance from slits to screen	path difference dark fringes light fringes
$\lambda \cong \frac{\Delta y d}{x}$	λ = wavelength Δy = space between fringes d = slit separation x = distance from slits to screen	Young's double-slit experiment

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$y_m \cong \frac{m\lambda L}{w}$ $(m = \pm 1, \pm 2, \pm 3 \ldots)$ $y_m \cong \frac{(m + \frac{1}{2})\lambda L}{w}$ $(m = \pm 1, \pm 2, \pm 3 \ldots)$	y_m = distance from fringe m to central bisector m = fringe order number L = distance to screen w = width of slit	single-slit interference destructive constructive
$\theta_{min} = \frac{\lambda}{w}$ $\theta_{min} = \frac{1.22\lambda}{D}$	θ_{min} = minimum angle for resolution λ = wavelength of light w = width of rectangular aperture D = diameter of circular aperture	resolution slit aperture circular aperture
$m\lambda = d \sin \theta$ $(m = 0, 1, 2, \ldots)$	m = fringe order number λ = wavelength of light d = distance between slit centres θ = angle from central bisector	diffraction grating bright fringes
$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$	c = speed of light μ_0 = permeability of free space ε_0 = permittivity of free space	speed of electromagnetic radiation
$E = hf$	E = energy of photon h = Planck's constant f = frequency of wave	energy of photon
$c = f\lambda$	c = speed of light f = frequency of wave λ = wavelength	wave equation
Equations in Unit 5 — Matter-Energy Interface		
$\Delta t = \frac{\Delta t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$ $L = L_o\sqrt{1 - \frac{v^2}{c^2}}$ $m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$ $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$	Δt = dilated time Δt_0 = proper time v = speed of object c = speed of light L = relativistic length L_0 = proper length m = relativistic mass m_0 = proper mass γ = gamma	time dilation length contraction mass increase variable substitution for simplicity
$E_k = mc^2 - m_oc^2$ $E_{k(max)} = hf - W$	E_k = relativistic kinetic energy m = relativistic mass m_0 = proper mass c = speed of light $E_{k(max)}$ = maximum kinetic energy of photoelectron h = Planck's constant f = frequency of electromagnetic radiation W = work function of metal	kinetic energy at relativistic speeds photoelectric effect

$p = \dfrac{h}{\lambda}$	p = momentum h = Planck's constant λ = wavelength	momentum of a photon
$\lambda = \dfrac{h}{mv}$	λ = wavelength m = mass v = velocity h = Planck's constant	de Broglie wavelength
$A = Z + N$	A = atomic mass number Z = atomic number N = number of neutrons	atomic mass number
$N = N_0\left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}}$	N_0 = original amount of radioactive material N = amount of radioactive material remaining after Δt Δt = time interval $T_{\frac{1}{2}}$ = half life	amount of radioactive material remaining
$v = \dfrac{2V}{Br}$ $\dfrac{m}{e} = \dfrac{2V}{v^2}$	v = speed of electron V = accelerating voltage B = magnetic field strength r = radius of trajectory m = mass of electron e = charge of electron	speed of electrons electron mass to charge ratio